Laplacian/RBF duality

j.p. lewis
Frostbite Labs

Frostbite Labs is EA's new skunkworks for developing future tech

May 18, 2016 | By Alex Wawro

This week Electronic Arts confirmed to investors that it has formed a dedicated "future tech" research division, Frostbite Labs, which is currently looking into (among other things) VR experiences, neural networks and machine learning.

While the division is surely focused on EA's proprietary Frostbite engine tech, this still means roughly 30-40 people across two offices (one in Vancouver, the other in Stockholm) are working on EA's dime to solve emerging game industry problems like: How do you create a believable "virtual human" for a VR game?

"How you're seen as a virtual human in that world is something we need to solve for," EA exec Patrick Söderlund said (according to Develop) as part of a presentation to investors about the new initiative.

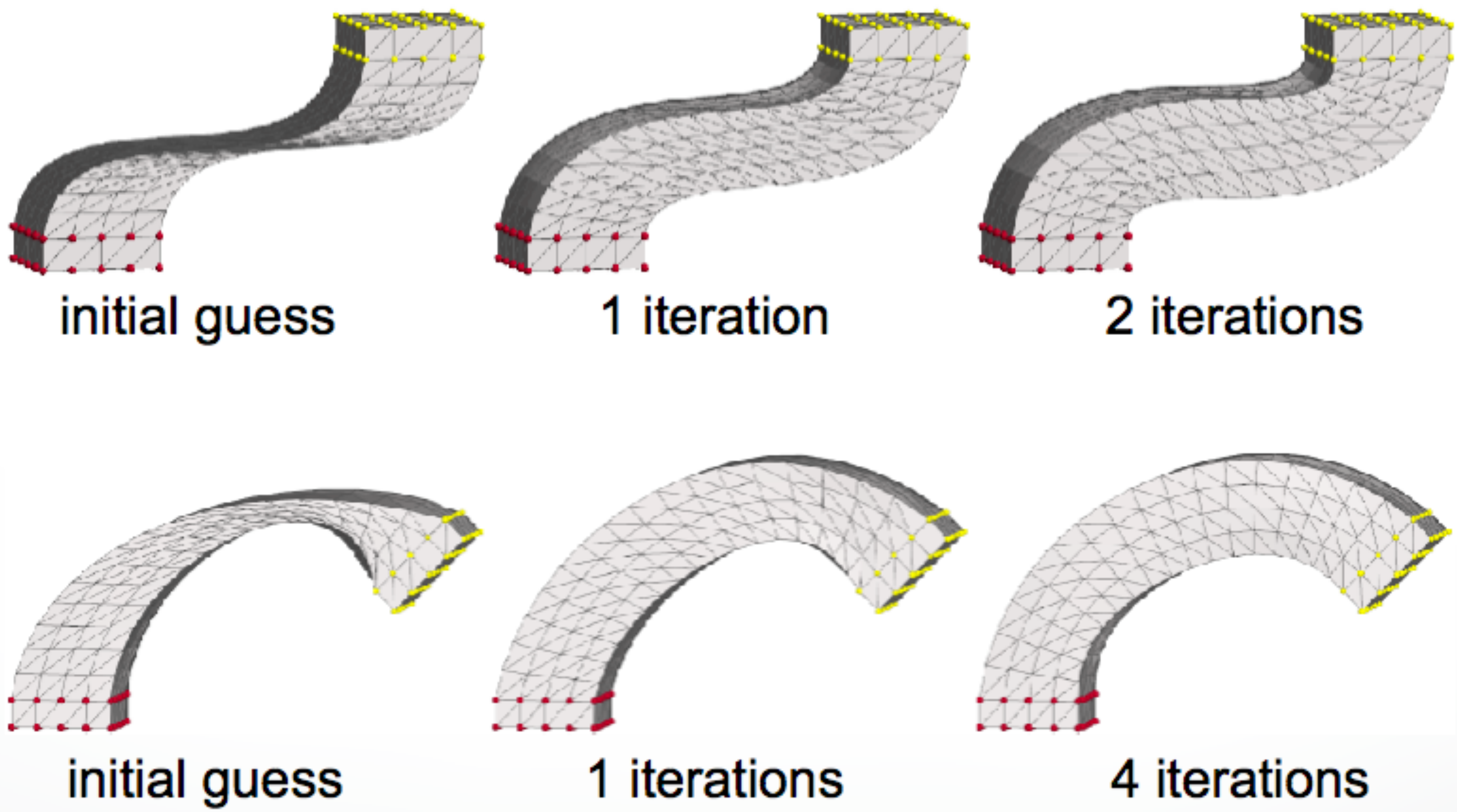
He went on to note that Frostbite Labs researchers are looking beyond VR at tech that could make the practice of game development easier. Using "deep learning" machine learning algorithms, for example, to

GAMASUTRA
The Art & Business of Making Games

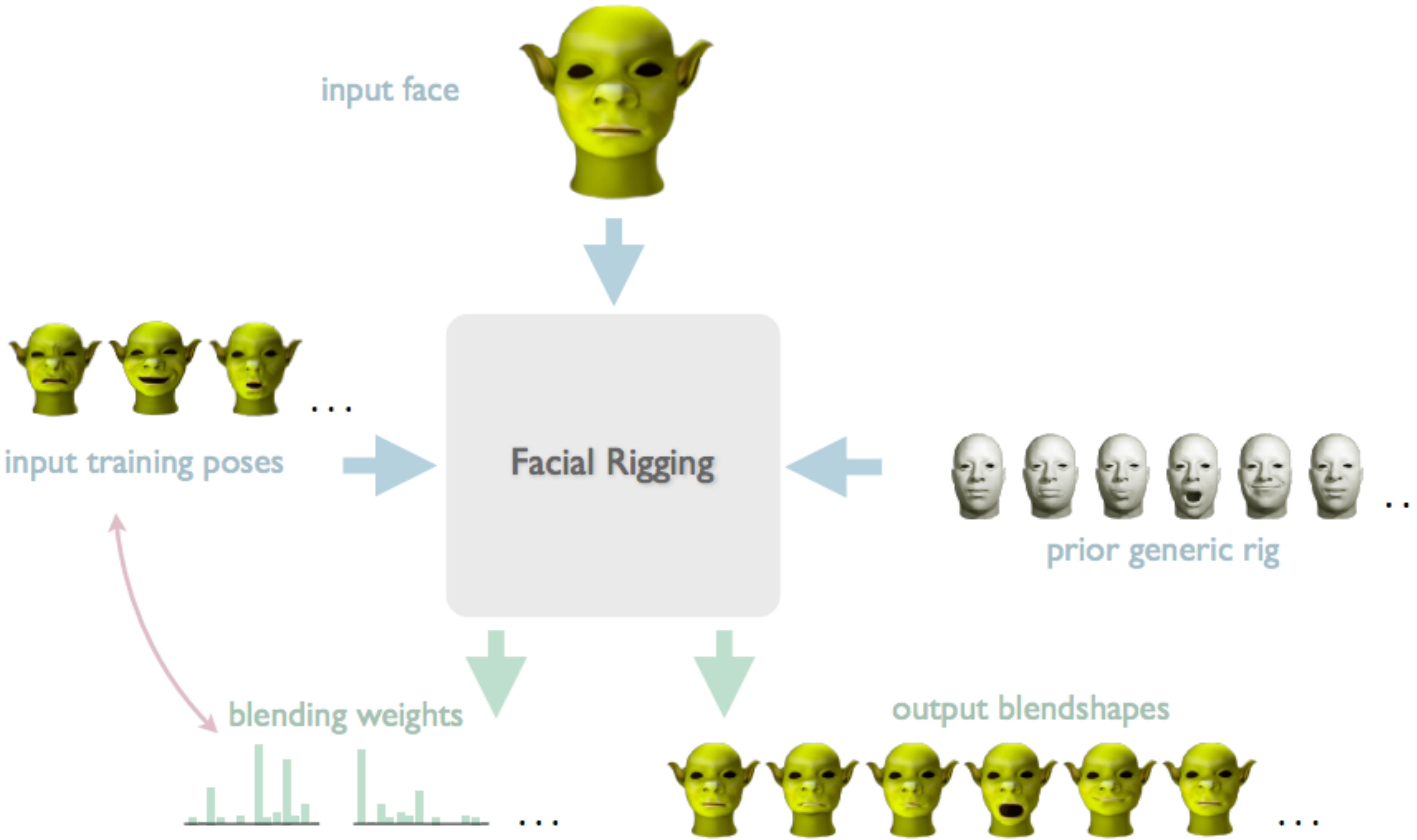
- mystery …

As-Rigid-As-Possible Modeling

- Start from naïve Laplacian editing as initial guess



Example Based-Facial Rigging



Skinning, RBF … meets Laplacians

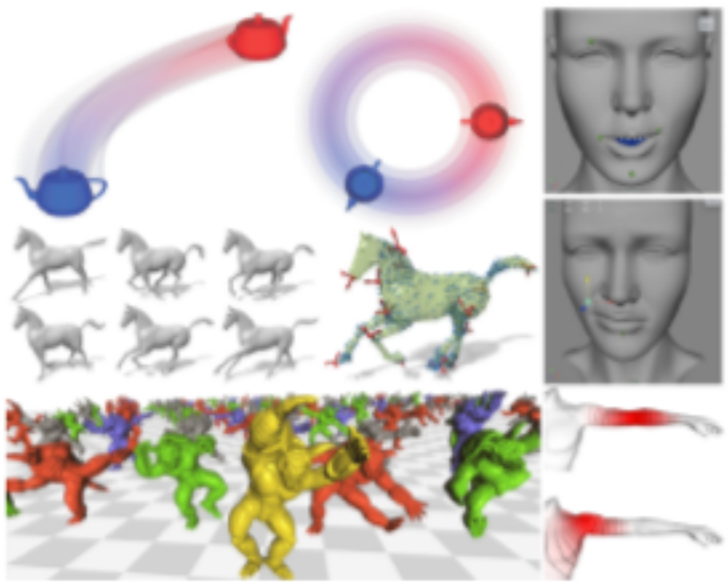
Skinning: Real-time Shape Deformation

skinning.org

ACM SIGGRAPH 2014 Course
ACM SIGGRAPH Asia 2014 Invited Course
Symposium on Geometry Processing 2015 Invited Course
International Geometry Summit 2016 Invited Course

Alec Jacobson
Columbia University
Zhigang Deng
University of Houston
Ladislav Kavan
University of Pennsylvania
J.P. Lewis
Victoria University, Weta Digital

SIGGRAPH Asia lecturer: Yotam Gingold
George Mason University



Course Materials

- Part I: Direct methods (Ladislav Kavan)
  Course notes | Slides
- Part II: Automatic methods (Alec Jacobson)
  Course notes | Course notes (low resolution) | Slides | Slides (167MB .pptx with videos)
- Part III: Example-based methods (JP Lewis)
  Course notes
- Part IV: Skinning decomposition (Zhigang Deng)
  Course notes | Course notes (low resolution) | Slides

Laplacian operator and RBF are "dual" (sometimes)

RBF "kernels":

3D thin plate spline	$\propto \lvert r \rvert$
2D thin plate spline	$\propto r^2 \log r$

where do these come from?

Green's function (very abstractly)

$\mathbf{Df} = \mathbf{b}$ abstract linear differential equation

$$\mathbf{f} = \mathbf{G}\mathbf{b}$$

$$\mathbf{D}\mathbf{G}\,\mathbf{b} = \mathbf{b}$$

$$\therefore \mathbf{G} = \mathbf{D}^{-1}$$

but: null space

$$\mathbf{Df} = \mathbf{b} \qquad \text{abstract linear differential equation}$$

change to discrete, 1D case

$$f[t] - f[t-1] \quad \approx \quad \text{derivative}$$

$$= (1, -1) \cdot (f[t], f[t-1])$$

$$\mathbf{D} = \begin{bmatrix} 1 & -1 & & & \\ & 1 & -1 & & \\ & & 1 & -1 & \\ & & & \cdots & \\ & & & & \end{bmatrix}$$

$$\min_{\mathbf{f}} \quad \|\mathbf{Df}\|^2 = \mathbf{f}^T\mathbf{D}^T\mathbf{Df}$$

$$\frac{d}{d\mathbf{f}} = 0 = 2\mathbf{D}^T\mathbf{Df}$$

the 1D Laplacian!

$$\mathbf{D}^T\mathbf{D} = \begin{bmatrix} & \cdots & & & & \\ 1 & -2 & 1 & & & \\ & 1 & -2 & 1 & & \\ & & 1 & -2 & 1 \cdots & \\ & & & & \vdots & \end{bmatrix}$$

$$\mathbf{L} \equiv \mathbf{D}^T \mathbf{D}$$

$$\mathbf{L}\,\mathbf{f} = \mathbf{b}$$

$$\text{represent} \qquad \mathbf{f} = \mathbf{G}\mathbf{b}$$

$$\text{substitute} \qquad \mathbf{L}(\mathbf{G}\mathbf{b}) = \mathbf{b}$$

$$\therefore \qquad \mathbf{G} = \mathbf{L}^{+}$$

G is our "RBF kernel"!

bug in wiki page? (check)

Green's function - Wikipedia

Secure https://en.wikipedia.org/wiki/Green%27s_function

gm w EA ET C AX MBlog Clk n gsoft w2 gs kernel Maya git Vic VL Safari Maya » Other Bookmar

Differential Operator L	Green's Function G	Example of application
∂_t^{n+1}	$\frac{t^n}{n!}\Theta(t)$	
$\partial_t + \gamma$	$\Theta(t)e^{-\gamma t}$	
$(\partial_t + \gamma)^2$	$\Theta(t)te^{-\gamma t}$	
$\partial_t^2 + 2\gamma\partial_t + \omega_0^2$	$\Theta(t)e^{-\gamma t}\frac{\sin(\omega t)}{\omega}$ with $\omega = \sqrt{\omega_0^2 - \gamma^2}$	1D damped harmonic oscillator
2D Laplace operator $\Delta_{2D} = \partial_x^2 + \partial_y^2$	$\frac{1}{2\pi}\ln\rho$	2D Poisson equation
3D Laplace operator $\Delta_{3D} = \partial_x^2 + \partial_y^2 + \partial_z^2$	$\frac{-1}{4\pi r}$	Poisson equation
		stationary

Laplacian vs RBF - speed

Laplacian^n	RBF
O(n) in number of unknowns	O(n^3) in number of **knowns**

(caricature slides)

(matrix slides)

Frostbite Labs

- internships
- collaboration
- jobs

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