Derivative & Integration from a Linalg POV

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— Damon Shelton

- function
- functional
- operator

continuous, discrete derivative

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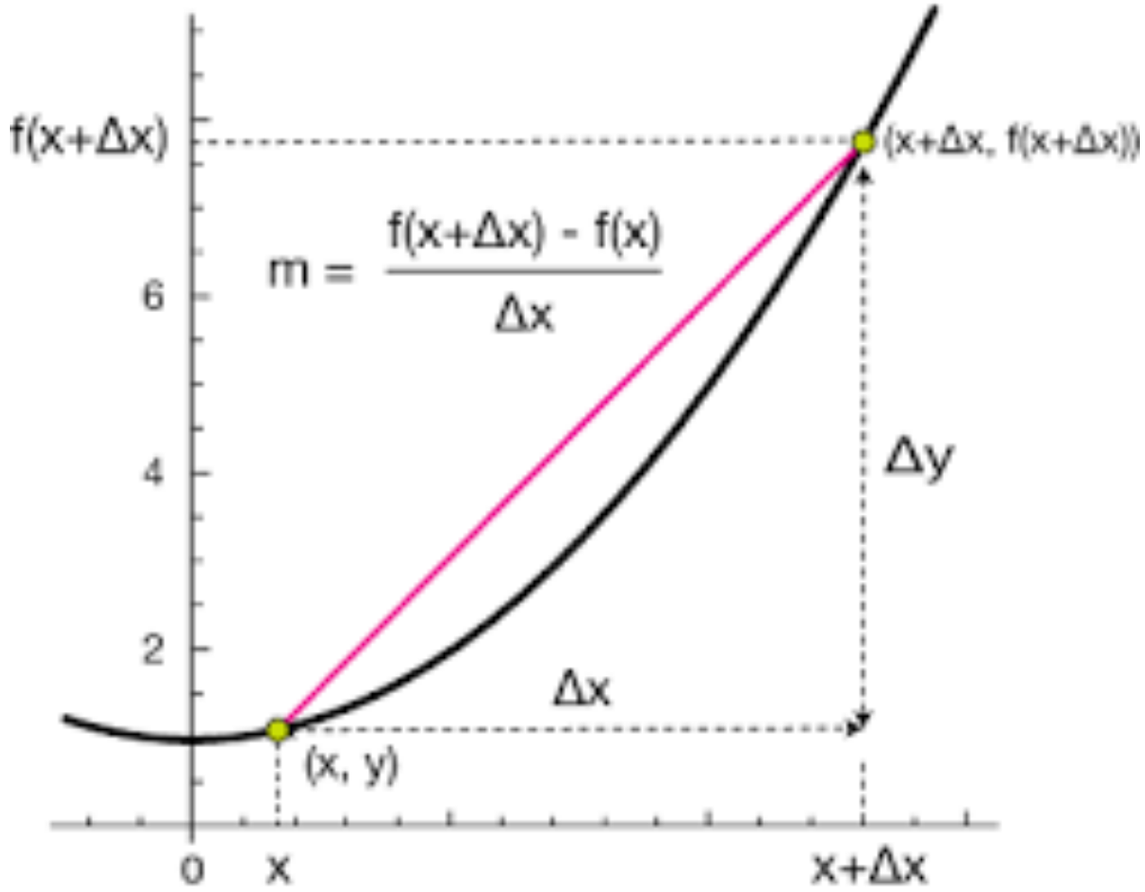
$$\lim_{\delta \to 0} \frac{f(x+\delta) - f(x)}{\delta}$$



continuous, discrete derivative

$$\text{continuous} \quad \lim_{\delta \to 0} \frac{f(x+\delta) - f(x)}{\delta}$$

$$\text{discrete} \quad f(x+1) - f(x)$$

$$\mathbf{Df} = \begin{bmatrix} -1 & 1 & & & & \\ & -1 & 1 & & & \\ & & -1 & 1 & & \\ & & & -1 & 1 & \\ & & & & \ddots & \end{bmatrix} \begin{bmatrix} f[0] \\ f[1] \\ f[2] \\ \cdot \\ \vdots \end{bmatrix}$$

```
D = [[ 1.  0.  0.  0.  0.  0.  0.  0.]
     [-1.  1.  0.  0.  0.  0.  0.  0.]
     [ 0. -1.  1.  0.  0.  0.  0.  0.]
     [ 0.  0. -1.  1.  0.  0.  0.  0.]
     [ 0.  0.  0. -1.  1.  0.  0.  0.]
     [ 0.  0.  0.  0. -1.  1.  0.  0.]
     [ 0.  0.  0.  0.  0. -1.  1.  0.]
     [ 0.  0.  0.  0.  0.  0. -1.  1.]]
```

```
np.dot(D.T,D) = [[ 2. -1.  0.  0.  0.  0.  0.  0.]
                 [-1.  2. -1.  0.  0.  0.  0.  0.]
                 [ 0. -1.  2. -1.  0.  0.  0.  0.]
                 [ 0.  0. -1.  2. -1.  0.  0.  0.]
                 [ 0.  0.  0. -1.  2. -1.  0.  0.]
                 [ 0.  0.  0.  0. -1.  2. -1.  0.]
                 [ 0.  0.  0.  0.  0. -1.  2. -1.]
                 [ 0.  0.  0.  0.  0.  0. -1.  1.]]
```

integral ~ running sum

```
[[ 1.  0.  0.  0.  0.  0.  0.  0.]
 [ 1.  1.  0.  0.  0.  0.  0.  0.]
 [ 1.  1.  1.  0.  0.  0.  0.  0.]
 [ 1.  1.  1.  1.  0.  0.  0.  0.]
 [ 1.  1.  1.  1.  1.  0.  0.  0.]
 [ 1.  1.  1.  1.  1.  1.  0.  0.]
 [ 1.  1.  1.  1.  1.  1.  1.  0.]
 [ 1.  1.  1.  1.  1.  1.  1.  1.]]
```

```
[[ 1.  0.  0.  0.  0.  0.  0.  0.]
 [ 1.  1.  0.  0.  0.  0.  0.  0.]
 [ 1.  1.  1.  0.  0.  0.  0.  0.]
 [ 1.  1.  1.  1.  0.  0.  0.  0.]
 [ 1.  1.  1.  1.  1.  0.  0.  0.]
 [ 1.  1.  1.  1.  1.  1.  0.  0.]
 [ 1.  1.  1.  1.  1.  1.  1.  0.]
 [ 1.  1.  1.  1.  1.  1.  1.  1.]]
```

```
[[ 1.  0.  0.  0.  0.  0.  0.  0.]
 [-1.  1.  0.  0.  0.  0.  0.  0.]
 [ 0. -1.  1.  0.  0.  0.  0.  0.]
 [ 0.  0. -1.  1.  0.  0.  0.  0.]
 [ 0.  0.  0. -1.  1.  0.  0.  0.]
 [ 0.  0.  0.  0. -1.  1.  0.  0.]
 [ 0.  0.  0.  0.  0. -1.  1.  0.]
 [ 0.  0.  0.  0.  0.  0. -1.  1.]]
```

the null space