## Derivative & Integration from a Linalg POV

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If you can't explain something to your grandmother, you don't really understand it.

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Damon Shelton

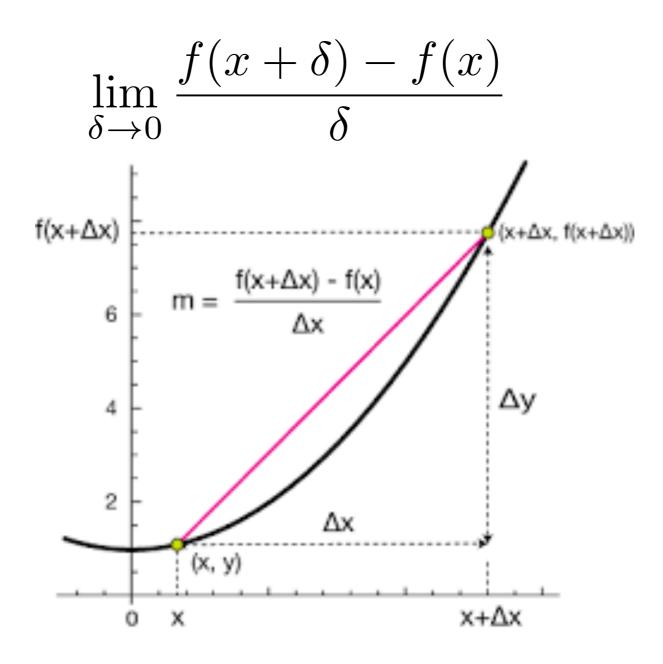
function

functional

operator

## continuous, discrete derivative

#### continuous, discrete derivative



### continuous, discrete derivative

continuous 
$$\lim_{\delta \to 0} \frac{f(x+\delta) - f(x)}{\delta}$$

discrete 
$$f(x+1) - f(x)$$

$$\mathbf{Df} = \begin{bmatrix} -1 & 1 & & & & \\ & -1 & 1 & & & \\ & & -1 & 1 & & \\ & & & -1 & 1 & \\ & & & & -1 & 1 \\ & & & & & \dots \end{bmatrix} \begin{bmatrix} f[0] \\ f[1] \\ f[2] \\ \vdots \end{bmatrix}$$

```
0. 0. 0.
                                 0.
                                     0.
                                        0.
                                              0.]
           D = [[ 1.
                                 0.
                          0. 0.
                    1.
                                     0.
                                              0.1
                [0.-1.1.0.0.0.
                                     0.
                                              0.1
                [ 0. 0. -1. 1.
                                 0.
                                     0.
                                          0.
                                              0.]
                [0. 0. 0. -1. 1.
                                     0.
                                              0.1
                [ 0. 0. 0. 0. -1. 1.
                                              0.]
                [0. 0. 0. 0. 0. -1. 1.
                [ 0. 0.
                          0. \quad 0. \quad 0. \quad 0. \quad -1. \quad 1.
                                              0.]
np.dot(D.T,D) = [[2.-1.0.0.0.]
                                      0. 0.
                [-1. 2. -1. 0. 0. 0. 0.
                                              0.1
                [0.-1.2.-1.0.0.0.
                                              0.1
                  0. \quad 0. \quad -1. \quad 2. \quad -1. \quad 0. \quad 0.
                                              0.1
                [0. 0. 0. -1. 2. -1. 0.
                                              0.]
```

 $0. \quad 0. \quad 0. \quad -1. \quad 2. \quad -1.$ 

[0. 0. 0. 0. 0. -1. 2. -1.]

[0. 0. 0. 0. 0. 0. -1. 1.]

0.1

# integral ~ running sum

```
0.
                              0.]
[[ 1.
      0.
          0. 0.
                  0.
                          0.
              0.
[ 1.
          0.
                  0.
                      0.
      1.
                          0.
                              0.]
      1.
          1.
              0.
                  0.
                      0.
[ 1.
                          0.
                              0.]
[ 1.
      1.
          1.
              1.
                  0.
                      0.
                          0.
                              0.]
          1.
      1.
              1.
                  1.
[ 1.
                      0.
                              0.]
                          0.
          1.
                              0.]
      1.
              1.
                  1.
                      1.
                          0.
[ 1. 1.
          1. 1. 1. 1. 1.
                              0.]
[ 1. 1. 1. 1. 1. 1. 1.
                              1.]]
```

```
0.
[ [ 1.
       0.
           0.
               0.
                   0.
                            0.
                                0.]
       1.
           0.
               0.
                    0.
                        0.
                            0.
                                0.]
           1.
               0.
                   0.
                        0.
                            0.
                                 0.]
           1.
               1.
       1.
                    0.
                        0.
                                 0.]
                            0.
       1.
           1.
               1.
                    1.
                        0.
                                 0.]
                            0.
      1.
           1.
               1.
                   1.
                       1.
                            0.
                                 0.]
      1.
           1.
              1.
                   1.
                       1.
                           1.
                                1.]
 [ 1. 1. 1. 1. 1. 1. 1.
```

```
0.
                  0.
                      0.
                           0.
        0.
                                0.
                                     0.]
[[ 1.
       1. 0.
                 0.
                      0.
                           0.
                                0.
                                     0.]
                  0.
                           0.
 [0. -1.
            1.
                      0.
                                0.
                                     0.]
      0. -1.
                 1.
                       0.
                           0.
                                0.
                                     0.]
        0.
           0. -1. 1.
                           0.
                                0.
                                     0.]
        0.
           0. \quad 0. \quad -1.
                          1.
                                0.
                                     0.]
             0. \quad 0. \quad 0. \quad -1.
        0.
                               1.
                                     0.]
 [ 0.
             0. \quad 0. \quad 0. \quad -1.
        0.
                                     1.]]
```

## the null space