Is the Fractal Model Appropriate for Terrain?

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Although it has been suggested that the fractal spectrum f^{-d} is to be preferred for theoretical reasons over other spectra for modeling many natural phenomena, theoretical and empirical arguments for considering other types of spectra will be mentioned.

Several researchers have concluded that the f^{-d} spectrum (or equivalently the "fractal dimension") is not adequate to describe natural phenomena of interest over all scales [5, 8]. Empirical studies of the possible fractal nature of various phenomena have also found that the fractal dimension is often different at different scales, although ranges of scales described by a constant dimension were found [4, 1]. In fact there is a relation between the fractal dimension D and the exponent d in the fractal power spectrum f^{-d} [12]

$$D = E + \frac{(3-d)}{2} \tag{1}$$

for a function of Euclidean dimension E. Thus, an analysis where the fractal dimension is allowed to vary with scale is equivalent to a form of spectral analysis. The 'piecewise fractal' approach is restricted however, in that nonsensical *negative* fractal dimensions or fractal dimensions greater than the dimension of the embedding space can result from spectral exponents that are physically quite reasonable. A piecewise fractal spectrum must be monotonically decreasing and hence cannot describe oscillatory phenomena such as waves. Ironically, the fractal dimension is often estimated from the power spectrum or autocorrelation function [1].

These arguments do not deny that the assumption of a power-law spectrum is a convenient analytical simplification. If "fractal analysis" is to be more than a procedure of fitting straight line segments through a spectrum curve plotted on log-log axes, however, it should be established that the attributed power law is accurate over an unexpectedly broad frequency interval, or that there are theoretical reasons for expecting this spectrum. The often-cited fractal interpretation of Richardson's coastline data has been questioned on these grounds because the data describe a fairly limited range of scales (less than two orders of magnitude in all but one case) and because there is no reason to expect the processes involved in very large scale (continent formation) and small scale coastline formation (e.g. erosion) to have similar statistics [9, 10]. It also appears that the nominal $1/f$ noises in electrical circuits may only approximate fractal behavior [6]).

Theoretical models predicting the fractal spectrum have not been forthcoming in most cases (an exception is the well established Kolmogorov $-5/3$ law in the statistical theory of turbulence [7]). In fact, the f^{-d} spectrum is *a priori* an unlikely choice for a theoretical model, since the integral of this function diverges, and consequently a phenomenon that strictly obeys this spectrum is nondifferentiable and has infinite characteristics (length, average power). The viability of this aspect of the fractal model was also questioned in the geophysical literature [3, 10].

Although some fractal practitioners state that the fractal model is only intended to apply over a limited range of scales, the global properties of self-similarity and non-differentiability are the distinguishing features of the fractal model. If these features are not required, one could adopt other spectra that fall off according to a power law over the desired range of scales. For example, the Markov-like spectra $(\alpha + f)^{-d}$ are commonly used as a "first-order fit" for many phenomena; these spectra are similar to the fractal spectrum at high frequencies yet do not diverge at low frequencies (the Markovian spectrum ($d = 2$) has a physical basis, e.g., in Langevin's equation for Brownian motion [13]).

One advantage of the fractal model over many others is that it generates detail at all scales. The general spectral modeling approach advocated here can also generate detail at all scales, without sacrificing differentiability and scale-

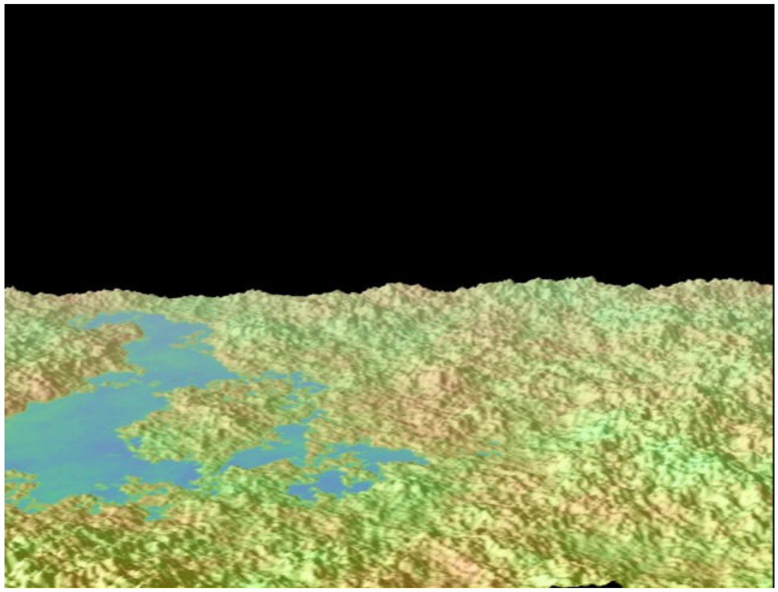

Figure 1: Fractal surface without power modification.

Figure 2: Non fractal (oscillatory) surface.

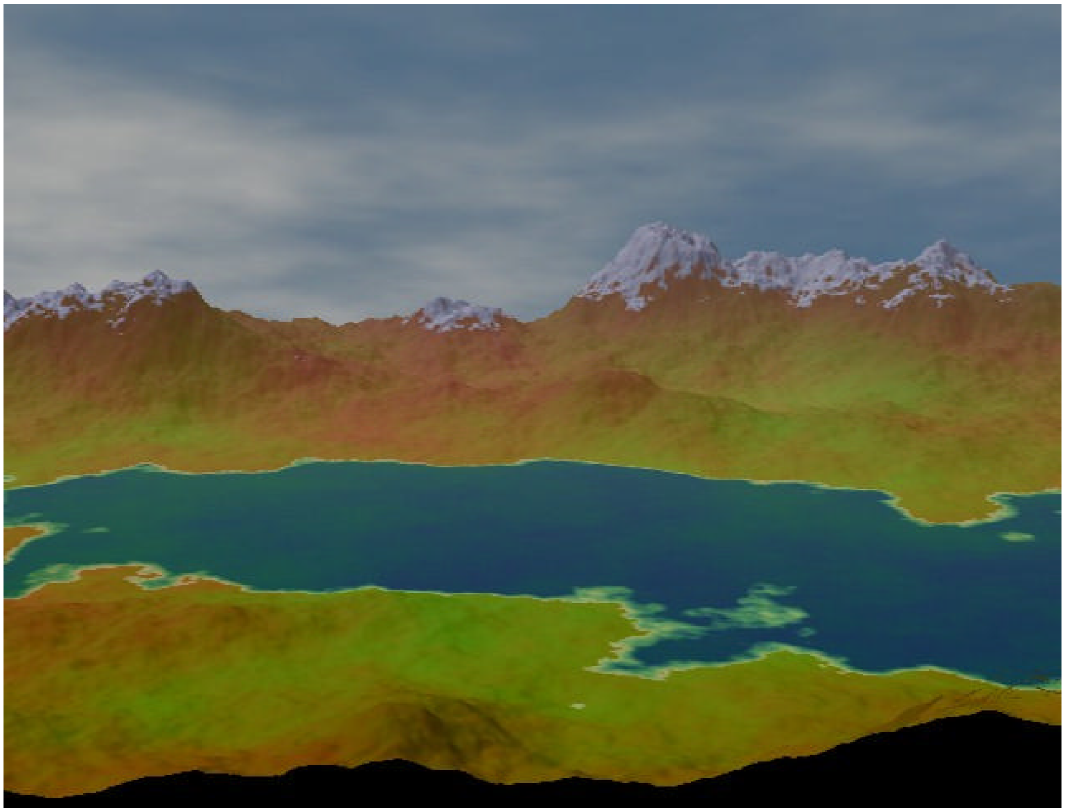

Figure 3: Power-modified fractal landsape.

dependent features. The nondifferentiability of the fractal models results in terrains that in the author's opinion appear too rough at small scales (e.g., Figure 1 and [11]). Most of the terrain figures accompanying this article were produced using two-dimensional variations of a "Markaussian" autocorrelation $R(\tau) = \exp(-|\tau|^{\alpha})$ with $1 < \alpha < 2$, which produces noises falling between the extremes of the Gaussian (too smooth) and Markovian (usually too rough) textures shown in Figure 6.

The desirability of scale-dependent features for visual modeling is also obvious: most visual phenomena provide at least an approximate sense of scale (atmospheric phenomena are perhaps the exception again). As an illustration, one would be surprised to find a terrain such as Figure 3 at one's feet, and the scale-dependent depressions in Figure 4 certainly do not detract from the realism of that terrain. The lack of scale-dependent features can be a practical problem in visual terrain simulation, since without such detail the observer cannot judge the distance to the "ground".

We conclude that, although a single descriptor such as the fractal dimension or spectrum exponent may be adequate for some classification purposes, the validity of the fractal model is not well established for many natural phenomena, and it is evident that the visual characteristics of many phenomena cannot be adequately differentiated using only this model. For example, it was found in [1] that an airport runway had a fractal dimension identical to that of some topographical data, the interpretation being that this was due to the attenuated low-frequency variation of the runway.

Spectral modeling with Gaussian processes permits the description of a variety of phenomena, including fractal noises as a special case. Important perceptual characteristics of the noise, such as scale, period of oscillation, and directional tendencies are directly reflected in the noise autocorrelation function. Other characteristics such as dominant scales of detail and the small-scale or high-frequency behavior of the noise are easier to specify in the frequency domain, using the intuitive interpretation of the spectrum as the amount of detail at each scale.

`These comments originally appeared in section 6.1 of the academic paper [2].`

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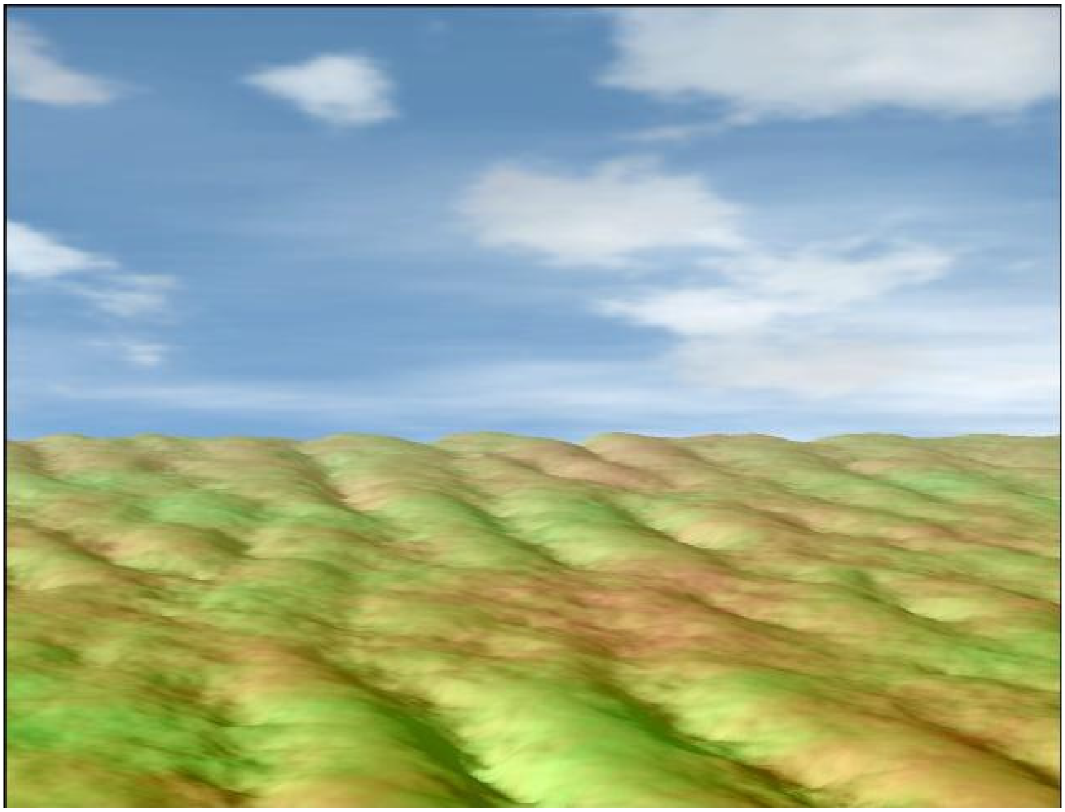

Figure 4: Non-fractal synthetic landscape.

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Figure 5: Non-fractal synthetic landscape.

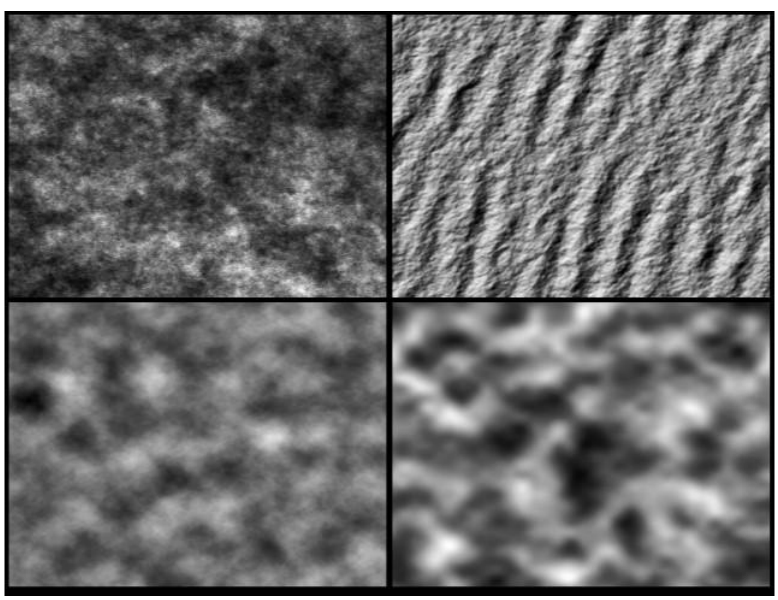

Figure 6: Several textures produced using the generalized subdivision technique. Clockwise from top-left: Markovian, oscillatory Markovian (shaded as an obliquely illuminated height field), Gaussian, and (isotropically) oscillatory "Markaussian" textures.