Lifting Detail from Darkness

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Disney The Secret Lab
Brightness-Detail Decomposition

- Separate detail by Wiener filter
- Represent brightness by membrane PDE
- “Unsharp masking 2.0”

Topics: Wiener filter, Laplace PDE, multigrid
Brightness-Detail Decomposition

- Modify detail, keep brightness. Example: texture replacement (subject to limitations of 2D technique)
- Modify brightness, keep detail. Example: 102 Dalmatians spot removal
Motivation: 102-Dalmatians
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"It turned out to be a huge job, much bigger than any of us had imagined."
"any spot that procedural could get rid of would help"...

No easy trick
No easy trick

Hypothetical spot luminance profile, blurred luminance (heavy), and unsharp mask (bottom).
No easy trick

Spot transition width is quite different on opposite sides of the same spot.
Wiener

- optimal linear estimator; for Gaussian data is optimal period.
- same principles can be applied as
  - filter
  - interpolator
  - predictor
- apply in frequency domain or spatially, recursive setting → Kalman
Wiener 1D

\[ y = x + n \]  \( x \): the unknown,  \( y \): observation

\[ \hat{x} = ay \]  \( \hat{x} \) is estimate. Find best \( a \)

\[ E \]
\[ E[ax + by] = aE[x] + bE[y] \]
\[ E[xn] = 0 \]

expectation operator

E is linear

noise not correlated with signal
Wiener 1D

\[ \min_a E[(\hat{x} - x)^2] \quad \text{Find } a \text{ that minimizes expected } \text{err}^2 \]
\[ \min_a E[a^2 y^2 - 2ayx + x^2] \quad \text{expand square, } \hat{x} \]
\[ \min_a E[a^2(x + n)^2 - 2a(x + n)x + x^2] \quad \text{expand } y \]

Expand products; \( E[xn] = 0 \):

\[ \min_a E[a^2(x^2 + 2xn + n^2) - 2a(x^2 + xn) + x^2] \]

\[ \frac{\partial}{\partial a} = 0 = 2aE[x^2 + n^2] - 2E[x^2] \quad \text{minimize} \]

\[ a = \frac{E[x^2]}{E[x^2 + n^2]} \]
Wiener principle

\[ a = \frac{E[x^2]}{E[x^2 + n^2]} \]

Signal variance divided by signal variance + noise variance.
How to use Wiener

Interpret “noise” as detail, “signal” as brightness.
\( \nabla^2 u = 0 \)  

\( (\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}) \)

- scattered interp for when there are lots of data (c.f. radial basis, other: invert \( N^2 \) matrix for \( N \) data points)
- minimizes integrated gradient-squared (roughness)
- minimizes small-deflection approx. to surface area
Membrane

Left - impulses in ‘L’ pattern. Black means no data, interpolate.

Right - membrane interpolation.
Membrane: minimize roughness

Roughness

$$R = \int |\nabla u|^2 du \approx \sum (u_{k+1} - u_k)^2$$

For a particular $k$:

$$\frac{dR}{du_k} = \frac{d}{du_k} [(u_k - u_{k-1})^2 + (u_{k+1} - u_k)^2]$$

$$= 2(u_k - u_{k-1}) - 2(u_{k+1} - u_k) = 0$$

$$u_{k+1} - 2u_k + u_{k-1} = 0$$

Or

$$\rightarrow \nabla^2 u = 0$$
Laplacian mask

1D:

1 -2 1

2D:

1
1 -4 1
1

1
Membrane as matrix eqn

\[ \nabla^2 u = 0 \]

rewrite \( \nabla^2 \) as matrix

\[ Mu = 0 \]

rows looks like:

\[
\begin{align*}
\cdots & 1, \ldots, 1, -4, 1, \ldots, 1, \ldots \\
\cdots & 1, \ldots, 1, -4, 1, \ldots, 1, \ldots \\
\cdots & 1, \ldots, 1, -4, 1, \ldots, 1, \ldots \\
\cdots & 1, \ldots, 1, -4, 1, \ldots, 1, \ldots \\
\cdots & 1, \ldots, 1, -4, 1, \ldots, 1, \ldots \\
\end{align*}
\]

\( M \) is huge - number of pixels in the region, squared! But \( M \) is sparse, five diagonal bands.
Membrane: relaxation

\( M u = 0 \) is too large to solve by conventional matrix inverse, solve by relaxation.

\[
    u_{k+1} - 2u_k + u_{k-1} = 0
\]

\[
    u_k \leftarrow 0.5(u_{k+1} + u_{k-1})
\]
Membrane: boundary conditions

For interpolation some $u_{r,c}$ are known/specified rather than free. In setting up the linear system, subtract these from both sides of the eq, so the known quantities move to the rhs.

$$\frac{1}{h^2} (u_{+0} + u_{-0} + u_{0+} + u_{0-} - 4u_{00}) = 0$$

Say $u_{+0}$ is known/fixed, then

$$\frac{1}{h^2} (u_{-0} + u_{0+} + u_{0-} - 4u_{00}) = -\frac{1}{h^2} u_{+0}$$
Membrane artifact
Membrane vs. Thin Plate

Left - membrane interpolation, right - thin plate.
Multigrid

\[ Ax = b \]

approximate solution \[ \hat{x} = x + e \]

\[ r = A\hat{x} - b \]
\[ r = Ax + Ae - b \]

but \[ Ax = b \] so \[ r = Ae \]

Residual has lower frequencies, so \( e \) can be solved at low res and subtracted from \( \hat{x} \) to give \( x \).
Multigrid results

- before: several minutes, > 1/2 gig of memory
- after: several seconds, memory not noticed
Algorithm steps

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Recovered fur
Recovered fur: detail
Versus hand cloning (manual)
Versus hand cloning (auto)
Versus hand cloning (manual), edge

Image has been sharpened
Versus hand cloning (auto), edge

Image has been sharpened
Other applications
Conclusions+Demo

- image alterations produced quickly with no artistic skill (cloning requires some paint skill)
- produces consistent effects across frames (cloning may ‘chatter’ unless done skillfully)